Ch. 17: Probability Models (Geometric and Binomial)

Opening question:

I have a game in which you spin a spinner for your prize. The spinner has 4 sections: Red 45%, Green 30%, Yellow 10% and Blue 15%. I spin 5 times.

X = # reds

$n = 5$

$p = 0.45$

$q = 0.55$

1- What is the probability of getting 3 reds?

2- What is the probability of getting 5 reds?

3- What is the probability of getting less than 3 reds?

4- What is the probability of getting at least 2 but no more than 4 reds?

X	0	1	2	3	4	5
$P(x)$						

How is this similar/different from GEOMETRIC MODELS?

Let's look at the 4 conditions:

- Outcomes: 2 : success/failure
- Probability of success: constant
- Number of trials: KNOWN
- Independent trials? yes

This type of model is...

BINOMIAL MODELS:

- Interested in the number of successes in a set number of trials
- 4 conditions that must apply:
  - 2 possible outcomes
  - Probability of success remains constant (called p)
  - Number of trials is SET (KNOWN)
  - Independent trials

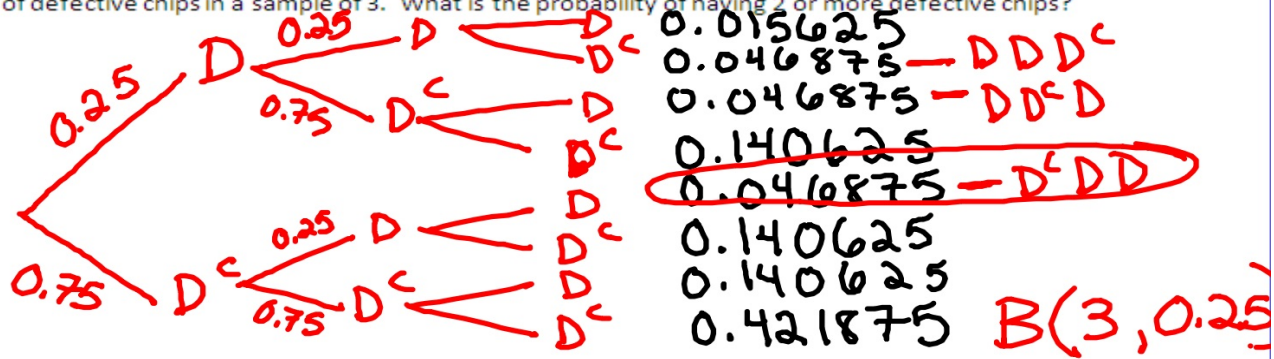
Notation: $B(n, p)$ $B(5, 0.45)$

$E(x) = \mu_x = n \cdot p$ $SD = \sigma_x = \sqrt{n \cdot p (1-p)}$ $\sqrt{n \cdot p \cdot q}$

$B(10, 0.25)$ 0.25×10

Binomial Probabilities

Example: Computer chips have a 25% chance of being defective. Create the probability distribution for X, if X is the # of defective chips in a sample of 3. What is the probability of having 2 or more defective chips?



X	0	1	2	3
P(X)	0.4219	0.4219	0.1406	0.0156

p. 393

How can we simplify this though?

Formula: $P(X=k) = (nCk)(p)^{k}(q)^{n-k}$

$P(X=2) = (3)(0.25)^{2}(0.75)^{1}$

Example: I am playing a game in which I have only a 39% chance of winning. I am playing 4 times. Create the probability distribution below:

X	P(X)	
0	$(4nCr0)(0.39)^0(0.61)^4$	0.13846
1	$(4nCr1)(0.39)^1(0.61)^3$	0.35409
2	$(4nCr2)(0.39)^2(0.61)^2$	0.33958
3	$(4nCr3)(0.39)^3(0.61)^1$	0.14474
4	$(4nCr4)(0.39)^4(0.61)^0$	0.02313

X	P(X)
0	0.13846
1	0.35409
2	0.33958
3	0.14474
4	0.02313

$n=4$

So let's answer some easy questions:

P(X=2) = 0.33958

P(X<2) = 0.49255

P(X≥3) = 0.16787

P(2≤X≤4) = 0.50745

Now let's look at changing the sample size to **10**, and answering similar questions:

X	P(X)
0	0.00713
1	0.04561
2	0.13121
3	0.22371
4	⋮
5	
6	
7	
8	
9	
10	

$B(10, 0.39)$

Now let's look at changing the sample size to **10**, and answering similar questions:

X	P(X)
0	0.00713
1	0.04561
2	0.13121
3	0.22371
4	0.2503
5	0.19203
6	0.10231
7	0.03738
8	0.00896
9	0.00127
10	0.000081

$L_2 = (10 \text{ nCr } L_1)(0.39 \wedge L_1)(0.61 \wedge (10 - L_1))$

P(X=9) =

P(X<4) =

P(X≥6) =

P(5≤X≤7) =

Would you want to answer these questions for a sample size of 30? Of 50? Of 100?

So we can use the calculator

For P(X=k) $P(X=7) = (10nCr7)(0.39)^7(0.61)^3$

- Use binompdf(n, p, k)
- k = # successes want
- pdf = prob. distr. fctn.

x	P(x)
1	⋮
2	
3	
⋮	

For P(X≤k) $P(X \leq 7)$

- Use binomcdf(n, p, k)
- k =
- Notice that is ONLY GIVES YOU ≤
- cdf =

$P(X<6)$

$P(X \leq 5)$

$P(X>3)$ — $1-P(X \leq 3)$

However you MUST still write _____

Example: John is taking archery. He has a 30% chance of hitting the target each time he shoots. He shoots 8 times

$B(8, 0.30)$

1) What is the probability that he hits the target 4 times?

$P(X=4) = \text{binompdf}(8, 0.30, 4) = 0.1361$

2) What is the probability that he hits the target 2 times or less?

$P(X \leq 2) = \text{binomcdf}(8, 0.3, 2) = 0.5518$

3) What is the probability that he hits the target at least 3 times?

$P(X \geq 3) = 1 - P(X \leq 2) = 0.4482$

4) What is the probability that he hits the target less than 5 times?

$P(X < 5) = P(X \leq 4) = \text{binomcdf}(8, 0.3, 4)$

5) What is the probability that he hits the target more than 6 times?

$P(X > 6) = 1 - P(X \leq 6)$

6) How many times do we expect him to hit the target? (average!)

$E(x) = n \cdot p = 2.4$

7) What is the standard deviation of the number of times he hits the target?

Try this example on your own: 150 businesses are sent mailings asking them to answer a survey question and send the mailing back. The probability of nonresponse is 55%.

1 & 2) What is the average number of businesses that WILL respond? std. dev?

3) What is the probability that 75 businesses will respond?

4) What is the probability that 60 businesses or less will respond?

5) What is the probability that 60 businesses or more will respond?

6) What is the probability that less than 60 businesses will respond?

7) What is the probability that greater than 60 businesses will respond?

8) What number of surveys would you have to send out if you wanted to be able to expect to get 90 back?

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